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NEAR-WALL TURBULENCE IN THE AXISYMMETRIC FLOW OF WEAK ACQUEOUS POLYMER SOLUTIONS

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The axisymmetric turbulent boundary layer is computed by using a finite-difference method and its fluctuation characteristics are determined on the basis of a generalized mixing-path hypothesis.

1. Investigation of turbulent exchange, including for polymer solution flows, is one of the important problems of applied modern hydromechanics. In addition to experimental investigations ([1, 2], say) it is expedient to produce a computational method that permits finding the fluctuation characteristics of different flows with a definite reliability. Such a method can be based on the generalized mixing-path hypothesis. Thus the possibility is shown in [3] of the possibility of describing the fluctuation characteristics in the near-wall layer of constant stress, pipes, and a flat plate boundary layer.

However, in the majority of cases the longitudinal pressure gradient and the three-dimensional nature of the flow influence the boundary layer development. Both these factors (the second not in the complete volume it is true but only in the ratio of the leakage-spread ratio of the retarded mass of the fluid) hold in an axisymmetric boundary layer whose analysis is comparatively simple because of its formal two-dimensionality. The analysis can here be performed within the framework of the conception of a thick layer by both an integral and finite-difference method [4], of which the latter permits the description of derivatives of the averaged velocity components and their associated tangential stress distributions in boundary layer sections with an accuracy sufficient for a subsequent calculation of the fluctuation characteristics.

The analysis of a thick axisymmetric turbulent boundary layer is performed in this paper for both a Newtonian fluid and for weak polymer solutions with the viscous-nonviscous interaction of the layer and wake with the external potential stream taken into account. Influence of the potential part of the flow on the boundary layer and wake parameters is taken into account in terms of the velocity distribution over their external boundary. To take account of the action of the viscous flow domain on the potential flow the latter is computed on a semi-infinite body formed by the external boundary layer boundary and the wake on which values of the normal velocity component found by means of the boundary layer and wake parameters are given. It is assumed that polymer admixtures influence the turbulent wake parameters only in terms of a change in the boundary layer characteristics at the site of layer and wake juncture, at the root extremity of the body. The action of the polymer admixtures on the potential flow is taken into account in terms of the change in layer and wake characteristics and the location of their external boundaries. The method in [5] is used to compute the wake and the method in [6], approved in [7] for the case of weak polymer solutions, is used to compute the potential domain.

The axisymmetric boundary layer equation in dimensionless form in Crocco variables can be written in the form [8]

$$\frac{\partial \omega}{\partial x} = \lambda_1 \frac{\partial^2 (\lambda_3 \omega)}{\partial u^2} + \lambda_2 \frac{\partial \omega}{\partial u} + \lambda_4 \quad (1)$$

with boundary conditions for the impenetrable surface

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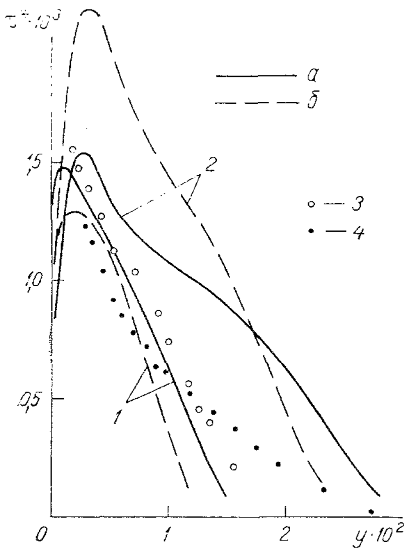

Fig. 1

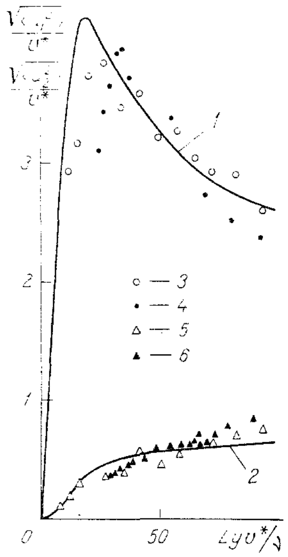

Fig. 2

Fig. 1. Tangential stress distribution: a) $\alpha = 0$; b) 10; 1) computation for $x_1 = 0.85$; 2) computation for $x_1 = 0.93$; 3) experiment [10], $x_1 = 0.85$; 4) experiment [10], $x_1 = 0.93$.

Fig. 2. Longitudinal and transvers fluctuation velocity component distribution in the near-wall part of the flow: 1, 2) computation; 3, 5) experiment [1]; 4, 6) experiment [2]; 1, 3, 4) $\sqrt{\langle u_1^2\rangle}/v^*$; 2, 5, 6) $\sqrt{\langle u_1^2\rangle}/v^*$.

$$v^* \omega \frac{\partial(\omega r)}{\partial u} = -\frac{r}{u_\delta}\frac{\partial u_\delta}{\partial x} \quad \text{for } u = 0, \qquad \omega = 0 \quad \text{for } u = 1, \tag{2}$$

where

$$\omega = \frac{\partial u}{\partial y};\ \lambda_1 = \frac{\omega^2}{ur};\ \lambda_2 = \frac{1-u^2}{uu_\delta};$$
$$\lambda_3 = (v^* + v_t^*)r,\ r = r_w + y\cos\theta; \tag{3}$$
$$\lambda_4 = \frac{\omega}{r}\left(\frac{\partial r}{\partial x} - \lambda_2\frac{\partial r}{\partial u} - \frac{r}{u_\delta}\frac{du_\delta}{dx}\right),$$

x, y, r_w and r are made dimensionless with respect to L.

To solve (1) under the boundary conditions (2) factorization according to an implicit finite-difference scheme is used, whose basis is the method of [9]. Equation (1) is represented as follows

$$\frac{\omega_m^{n+1} - \omega_m^n}{\Delta x} = (\lambda_1)_m^{n+\frac{1}{2}} \frac{(\lambda_3\omega)_{m+1}^{n+\frac{1}{2}} - 2(\lambda_3\omega)_m^{n+\frac{1}{2}} + (\lambda_3\omega)_{m-1}^{n+\frac{1}{2}}}{(\Delta u)^2} + (\lambda_2)_m^{n+\frac{1}{2}} \frac{\omega_{m+1}^{n+\frac{1}{2}} - \omega_{m-1}^{n+\frac{1}{2}}}{2\Delta u} + (\lambda_4)_m^{n+\frac{1}{2}}, \tag{4}$$

where n and m are the number of the computational grid section in x and u, Δx and Δu are the integration steps in x and u. The values of λ_1, λ_2, λ_4, ω and $\lambda_3\omega$ at the half-integer points n + 1/2 are determined in terms of the corresponding quantities at integer points n and n + 1

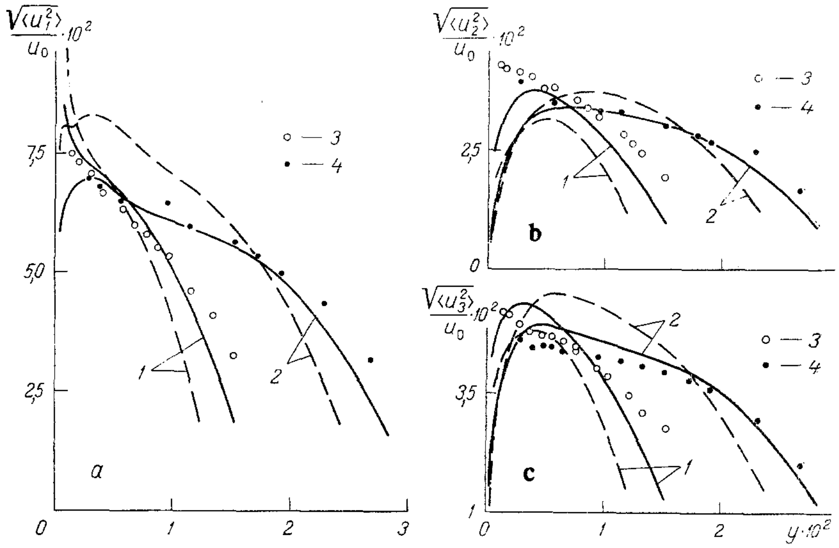

Fig. 3. Distribution of the r.m.s. values of the longitudinal (a) transverse (b), and transversal (c) fluctuation velocity components (notation the same as in Fig. 1).

of the grid by linear interpolation. Successive approximations in each step in x are used to compute the boundary layer. In a first approximation $\omega_m^{n+1} = \omega_m^n$ is taken in a given x section. The values of ω^{n+1} are refined during the factorization until sufficient accuracy in determining the value of ω on the streamlined surface is assured.

To determine the turbulent stresses, the mixing path hypothesis is used according to which

$$\nu_t^* = l^2 \omega, \tag{5}$$

where ℓ is represented in the form [4, 8]

$$l = l_0 l_1, \; l_1 = 1 - \exp\left(-\frac{y}{A\nu^* U_\delta}\sqrt{\frac{\tau_0}{\rho}}\right),$$

$$l_0 = \begin{cases} \varkappa y \text{ for } y < 0.5\,\delta_1, \\ -0.5\,\varkappa y^2/\delta_1 + 1.5\,\varkappa y - 0.125\,\beta\delta \text{ for } 0.5\,\delta_1 \leqslant y \leqslant 1.5\,\delta_1, \\ \beta\delta \text{ for } 1.5\,\delta_1 < y \leqslant \delta, \end{cases} \tag{6}$$

$$\beta = \frac{\xi}{\delta}\frac{\delta - \delta_1}{U_\delta - U_1}\sqrt{\frac{\tau_1}{\rho}}, \quad \delta = \int_0^{0.99} \omega^{-1}\,du, \; \delta_1 = \beta\delta/\varkappa,$$

$\varkappa = 0.4$ and $\xi = 0.75$ are constants governing the turbulent exchange in the inner and outer parts of the boundary layer; the value of the tangential stress τ_0 on the outer viscous sublayer boundary is found from the Prandtl equation written for y = 0

$$\tau_0 = \tau_w \frac{r_w}{r_0}\left(1 - \rho U_\delta \frac{dU_\delta}{dx}\frac{y_0}{\tau_w}\right),$$

and the location y_0 of this boundary from the condition $y_0\sqrt{\tau_w} = 9\nu^* U_\delta \sqrt{\rho}$. The value of the parameter A in an ordinary fluid is assumed constant $A = A_0 = 26$. The parameters entering into (6) are refined in a given layer section during iteration.

Taking into account that polymer admixtures act on the turbulent exchange principally in direct proximity to the wall, their influence can be determined in terms of the damping factor A that will depend on the kind of polymer, its concentration in the solution, and certain hydrodynamic characteristics of the flow. This dependence can be found, say, by using the Meyer correlation, by using the relation between A and a change in the additive constant in the logarithmic velocity profile by the quantity ΔB [4].

$$A = A_0 = 26 \text{ for } v^* < v_0^*,$$
$$A = A_0[5(v^*/v_0^*)^{\alpha/40\sqrt{2}} - 4] \text{ for } v^* \geq v_0^*, \quad (7)$$

where $v^* = \sqrt{\tau_w/\rho}$ is the dynamic velocity, v_0^* is its critical value corresponding to the beginning of the polymer effect, and α is the Meyer parameter. The quantities v_0^* and α are empirical constants whose values depend on the kind of polymer and its mass concentration c in the solution. Thus $v_0^* = 0.023$ m/sec and $\alpha = 10$ correspond approximately to a polyethylene oxide solution WSR = 301 for $c = 10^{-5}$. For these values the boundary layer of a body of revolution that is a combination of an ellipsoid and a cone with a total elongation of 6.21 was analyzed for a Reynols number $1.26 \cdot 10^6$, that corresponds to conditions for carrying out an experiment [10]. The profiles of certain of the tangential stresses obtained in the analysis are displayed in Fig. 1, where their agreement with experimental data in the case of an ordinary viscous fluid is quite satisfactory. Results of computations of profiles of average velocity projections and certain integral characteristics of the layer are presented in [4].

Introduction of the polymer admixtures in all sections for $x_1 > 0.7$ results in an increase in the mixing length in the outer part of the layer. Up to $x_1 = 0.85$ the presence of a dissolved polymer in the flow yields a diminution in the tangential stresses in the whole boundary layer thickness. For $x_1 > 0.85$ the polymer admixtures diminish τ in the viscous sublayer and the buffer domain and in the region of the outer boundary layer boundary but increase then in the central part of the layer, the domain of its turbulent core.

2. To describe the r.m.s. values of the fluctuation velocity $\sqrt{\langle u_i^2 \rangle}$ their representation [3] in terms of the mixing length ℓ_i, can be used, which is in dimensionless form

$$\sqrt{\langle u_i^2 \rangle}/U_0 = l_i \omega u_\delta, \quad (8)$$

where the subscripts i = 1, 2, 3 correspond to the longitudinal, transverse, and transversal directions

$$l_i = l_0 l_{1i},$$
$$l_{1i} = A_i\left[1 - B_i \exp\left(-C_i \frac{y}{Av^* U_\delta}\sqrt{\frac{\tau_0}{\rho}}\right)\right], \quad (9)$$

where the values of ℓ_0 are calculated from the relationships (6) while the coefficients A_i, B_i and C_i govern the singularity of the fluctuation velocity distribution in direct proximity to the wall and are taken to be equal to [3]: $A_1 = 2$, $A_2 = 1.1$, $A_3 = 1.5$, $B_1 = 0.625$, $B_2 = 1$, $B_3 = 0.99$, $C_1 = 1$, $C_2 = 0.5$, $C_3 = 0.7$. The results of computations of the longitudinal and transverse fluctuation velocity components for the near-wall part of the flow for A = 125 are compared with measurement data [1 and 2] in Fig. 2 and display satisfactory agreement between the experimental and computational quanties, which permits utilization of the proposed method of computing the fluctuation velocities for their determination in boundary layers in an ordinary viscous fluid and in weak aqueous polymer solutions.

Computation of the flow around a body of revolution from [10] were performed to analyze the influence of polymer admixtures on the near-wall turbulence characteristics in the case of gradient flow in an axisymmetric boundary layer. The results of these computations are displayed in Fig. 3 for two layer sections, $x_1 = 0.85$ and $x_1 = 0.93$. The dimensionless longitudinal pressure gradient dC_p/dx_1 in these sections, where the pressure coefficient is $C_p = 2(p - p_0)/(\rho U_0^2)$, equals 0.74 and 0.40, respectively. Comparison of the computation results with test data [10] in the case of ordinary viscous fluid flow around a body in the presence of a longitudinal pressure gradient shows satisfactory agreement.

The presence of polymer admixtures in the flow results in an increase in the longitudinal fluctuation velocity component in the area of the wall and its diminution in the outer part of the layer. Up to the section with $x_1 \approx 0.9$ both the transverse and the transversal fluctuation velocity components diminish in the whole layer thickness upon insertion of the polymer. For $x_1 > 0.9$ these fluctuation velocities in the polymer solution diminish at the wall and in the outer part of the layer while they grow somewhat in the turbulent core zone as compared with corresponding quantities in an ordinary fluid. Such a complex nature of the action of polymer admixtures on the fluctuation characteristics of the stream is explained by their strong influence in direct proximity to the wall, by diminution of the boundary layer thickness upon their insertion, and by the presence of a longitudinal pressure gradient and diminution of the body radius in the region of its root extremity. Attention is turned to the fact that sufficiently significant fluctuating velocities still exist in the stream on the outer boundary layer boundary, i.e., for $y = \delta$ ($u = 0.99$).

NOTATION

x, y are dimensionless longitudinal and transverse coordinates in the boundary layer; r_w is the dimensionless body radius; x_1 is the dimensionless coordinate along the body axis of symmetry; L is the body length; y_0 is the dimensionless viscous sublayer thickness; r_0 is the radius corresponding to the outer viscous sublayer boundary; δ is the dimensionless boundary layer thickness; δ_1 is the dimensionless thickness of the inner part of the layer; ℓ, ℓ_i are dimensionless mixing paths; u is the dimensionless longitudinal average velocity component; i_i are fluctuation velocity components'; U_δ is the velocity on the outer boundary layer boundary; U_0 is the free-stream velocity; $u_\delta = U_\delta/U_0$, U_1 is the velocity at $y = \delta_1$; ν is the kinematic viscosity; $\nu^* = \nu/(U_0 \cdot L)$ is the dimensionless viscosity; ν_t^* is the dimensionless turbulent viscosity; $\varkappa$, ξ, A_0 are turbulence constants; β, A are parameters governing the turbulent exchange in the outer and near-wall parts of the boundary layer; A_i, B_i, C_i are coefficients governing the fluctuation velocity distribution in the wall area; τ is the tangential stress; τ_i is its value at $y = \delta_1$; τ_0 is its value on the outer viscous sublayer boundary; τ_w; is its value on the wall, $\tau^* = \tau/\rho U_0$); n, m are numbers of computational grid sections in x and u, respectively; θ is the angle between the axis of the body of revolution and the tangent to its meridian section; λ_i are coefficients in the equations of motion written in Crocco variables; ρ is the fluid density; c is the polymer mass concentration in the solution; $\sqrt{\langle u_i^2 \rangle}$ is the r..m.s value of the fluctuation velocity components, i = 1, 2, 3; α is the Meyer parameter; p is the pressure; p_0 is the pressure at infinity; C_p is the pressure coefficient.

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